Agglomeration and comparative advantage in vertically-related firms

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Abstract

This paper models, in game-theoretical terms, the location of two vertically-linked monopolistic firms in a spatial economy formed by a large, high labor cost country and a relatively small, low labor cost country. It is found that the decrease in transport costs shifts firms towards the low production cost country. This process takes two different forms: in labor-intensive industries it leads to spatial fragmentation; in industries with strong input-output relations, agglomerations are conserved, although they shift toward the low labor cost country.

Keywords: Location; Intermediate goods; Agglomeration; Comparative advantage.

JEL classification: F10, F12, R30

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1. Introduction

The general decrease in transport costs has caused a shift in productive activity away from countries with large markets towards countries with low labor costs. This process has two different forms depending on the industry involved. In labor-intensive industries (such as the textile industry), with a low intensity of vertical linkages, it leads to spatial fragmentation. In this case, production is located in a low-cost country, while distribution and design are placed close to the majority of the consumers. By contrast, in sectors with strong input-output relations, such as the engineering sectors (aerospace, car, pharmaceuticals, electronics), the agglomerated pattern is maintained, but its location shifts toward a low production cost country. This paper provides a theoretical rationalization for these trends.

It is widely acknowledged that, when choosing to locate, firms face a trade-off between the location where access to consumers is maximized (usually the central point of the market) and the location where the firm's production costs are minimized. MAYER (2006) dealt with this problem in the context of a duopoly in a bounded linear space, where consumers are uniformly distributed and where, by contrast, the distribution of unit production costs is non-uniform. He concluded that with a globally convex distribution of production costs, there will be an agglomerated equilibrium of locations that is intermediate between the central point of the market and the minimum production cost point. He added that the equilibrium locations of firms will be closer to the minimum production cost point than to the central point, because a deviation from the former point would cause a loss to *all* consumers, whereas a deviation from the central point would harm *some* consumers, while benefiting other consumers. He also concluded that a fall in the unit transport costs of the consumer good would shift equilibrium locations toward the point of minimum production costs.

Usually, differences in production costs across locations follow from the fact that the supply of an input is localized, so that the firm not only has to pay the price of the input at its source but also has to transport it over the distance between the firm's location and the input site. MAYER (2000) explicitly considers this cause of spatial heterogeneity in production costs. However, the localized input is often an intermediate good produced by upstream firms. Hence the location of the input is endogenous and interdependent with the location of the consumer good firms. HWANG and MAI (1989) model this interdependence through a two-stage game involving two players that are successive monopolists. In the first stage, the upstream and the downstream firms simultaneously select locations in an interval whose left boundary is a "port", through which a raw material is imported, and whose right boundary is a "market" in which all consumers locate. In the second stage, the firms set mill prices for the intermediate good and for the consumer good. Subgame perfect equilibrium locations are derived for the firms, depending on the unit transport costs of the three goods (raw material, intermediate good and final good) and on the input-output coefficients. This model suffers from the limitation that the source of the primary input is, by assumption, distinct from the location of the consumers.

AMITI (2003) overcomes this limitation in the sense that she presents a general equilibrium model with two countries (Home and Foreign) which are each locations of both consumers and firms. There are two vertically-related industries, Upstream and Downstream, which both operate under monopolistic Dixit-Stiglitz competition. The industries use two primary factors, labor and capital, in different proportions: upstream firms are capital-intensive, while downstream firms are labor-intensive. The countries differ in terms of their factor endowments, so that Home is abundant in capital while Foreign is abundant in labor. Besides primary factors, each downstream firm uses a composite intermediate good made by the products of each upstream firm, as in ETHIER (1982).

Apart from the case of autarky, where upstream and downstream firms divide evenly between the two countries in order to serve the local consumers, there are two possibilities. If transport costs are intermediate, all the firms (upstream and downstream) agglomerate in one country, and the downstream industry supplies the other country in manufactured goods through exports. Agglomeration occurs in the Foreign (capital-abundant) country if the transport costs of the intermediate good are low enough in relation to the transport costs of the final good. Agglomeration takes place in the Home (labor-abundant) country if the transport costs of the intermediate good are high enough and the existence of multiple locational equilibria is possible. Finally, if both types of transport cost are low enough, the upstream and downstream firms locate in different countries, according to comparative advantage, and a fragmented equilibrium emerges. However, AMITI (2005) does not shed enough light on the basic trade-off that firms incur between production costs (which are mainly felt by upstream firms) and market access (which is mainly felt by downstream firms). The reason is that she focuses on the allocation of each production stage to the country that is abundant in the factor (capital or labor) used more intensively by that production stage.

In this paper, a partial equilibrium, game-theoretical model in the spirit of HWANG and MAI (1989) is used, although it relates to a case where two countries are both locations of consumers and firms. The countries are asymmetric both in market size and unit production costs, i.e. the country with the higher number of consumers also has higher production costs. There is a successive monopoly, where an upstream firm uses labor to manufacture an intermediate good. This input is transformed by a downstream firm into a final good that the firm then sells to final consumers. The locational pattern depends on the interaction of unit labor costs, vertical linkages and market access. An exact and detailed definition of locational equilibria in the space of two parameters (intensity of vertical linkages and transport cost) is produced, while the differentials in unit production costs and market size are accounted for through an adequate specification of parameters. The results confirm the position of AMITI (2005) as far as the occurrence of spatial fragmentation in the upstream and downstream stages is concerned, but they differ from her work in other respects, since factor intensity does not play a major role here. In this paper, agglomeration occurs for high transport costs (although in multiple locations), instead of dispersion.

In section 2, a model for the location of vertically-linked firms is presented. In section 3, the main conclusions are drawn.

2. The model

2.1. Assumptions

A spatial economy is defined by the following assumptions:

1. There are two countries, labeled Home (H) and Foreign (F). The number of consumers in H is higher than in F: $n_h > n_f$. The distance between H and F is normalized to 1. The distance between two points inside the same country is zero.

2. Each consumer has a linear demand function $q = a - bp$, where p is a delivered price.

3. There are two vertically-related firms. The upstream firm U transforms c_u units of labor into one unit of an intermediate good. The downstream firm D uses α units of the intermediate good and c_d units of labor to produce one unit of the final product that is sold to consumers. Firm U is more labor intensive than firm D, so that we have $c_u > c_d$.

4. The intermediate good has a transport cost τ and the final good has a transport cost t. These costs vary in proportion, following the evolution of the general transport infrastructure.

5. Each firm transports and delivers its product to its customers. Firm D sets discriminatory prices p_h, p_f in each country, while firm U sets a delivered price k for the intermediate good.

6. Country F is more labor-abundant than country H, so that the (parametric) wages are such that $w_h > w_f$.

2.2. The structure of the game

The game has two players, namely the firms U and D, and three stages:

First stage Firms U and D simultaneously select locations $x_u, x_d \in \{H, F\}$.

Second stage Firm U sets a delivered price k for the intermediate good.

Third stage Firm D sets discriminatory prices p_h, p_f for the consumer good.

The payoff (profit) functions of the firms are:

$$\begin{aligned}
\pi_d\left(p_h, p_f, k, x_u, x_d\right) &= n_h\left(a - bp_h\right)\left[p_h - \alpha k - c_d w_{xd} - td\left(x_d, H\right)\right] + \\
&\quad + n_f\left(a - bp_f\right)\left[p_f - \alpha k - c_d w_{xd} - td\left(x_d, F\right)\right] \qquad (1) \\
\pi_u\left(p_h, p_f, k, x_u, x_d\right) &= \alpha\left[n_h\left(a - bp_h\right) + n_f\left(a - bp_f\right)\right] \cdot \\
&\quad \cdot \left(k - \tau d\left(x_d, x_u\right) - c_u w_{xu}\right) \qquad (2)
\end{aligned}$$

where $d(,)$ is the distance function, and w_{xd} and w_{xu} are the parametric wage rates in the locations of the downstream and the upstream firms, respectively.

In order to concentrate our attention on the parameters that express the intensity of vertical linkages (α) and the level of transport costs (t), the following values are assigned to the parameters:

$$\begin{aligned}
n_h &= 1.3 > n_f = 1 \text{ (Country } H \text{ is larger than Country } F\text{)} \qquad (3) \\
c_u &= 1 > c_d = 0 \text{ (Upstream production} \\
&\quad \text{is more labor-intensive than downstream production)} \\
w_h &= 0.3 > w_f = 0.1 \text{ (Country } F \text{ is} \\
&\quad \text{more labor-abundant than country } H\text{)} \\
a &= b = 1 \\
t &= \tau \text{ (Transport costs of the intermediate good} \\
&\quad \text{and of the final good vary in proportion)}
\end{aligned}$$

With these specifications, 1 and 2 become

$$\begin{aligned}\pi_d(p_h,p_f,k,x_u,x_d) &= 1.3(1-p_h)[p_h-\alpha k-td(x_d,H)]+ \\ &\quad +(1-p_f)[p_f-\alpha k-td(x_d,F)] \qquad (4)\\ \pi_u(p_h,p_f,k,x_u,x_d) &= \alpha[1.3(1-p_h)+(1-p_f)][k-td(x_d,x_u)-w_{xu}] \qquad (5)\end{aligned}$$

The payoff matrix of the location (first-stage) game can be expressed by

$$\begin{array}{ccccc} & & \text{Downstream} & \\ & & H & F \\ \text{Upstream} & H & \pi_u(H,H),\pi_d(H,H) & \pi_u(H,F),\pi_d(H,F) \\ & F & \pi_u(F,H),\pi_d(F,H) & \pi_u(F,F),\pi_d(F,F)\end{array} \qquad (6)$$

2.3. Solving the game

In order to find a subgame perfect equilibrium, each subsequent subgame that begins in a cell of the payoff matrix 6 is solved by backward induction, yielding profits $\pi_u(\alpha,t)$ and $\pi_d(\alpha,t)$ that depend only on the intensity of vertical linkages and on transport costs. The details of these calculations are explained in the Appendix. The profits in the first-stage game are:

Outcome $x_u = x_d = H$

$$
\begin{aligned}
\pi_d(H,H) &= (0.075\,\alpha + 0.391\,31t - 0.25)^2 + && (7)\\
&\quad +1.3\,(0.075\,\alpha - 0.108\,70t - 0.25)^2 \\
\pi_u(H,H) &= a\left(a\left(\frac{0.250\,00}{a}t - \frac{0.575}{a} - 0.172\,5\right) - 0.5t + 1.15\right)\cdot && (8)\\
&\quad \cdot\left(\frac{0.5}{a} - \frac{0.217\,39}{a}t - 0.15\right)
\end{aligned}
$$

Outcome $x_u = H, x_d = F$

$$
\begin{aligned}
\pi_d(H,F) &= (0.075\,\alpha - 0.141\,31t + 0.25\alpha t - 0.25)^2 + && (9)\\
&\quad +1.3\,(0.075\,\alpha + 0.358\,70t + 0.25\alpha t - 0.25)^2 \\
\pi_u(H,F) &= \alpha\left(\frac{0.5}{\alpha} - 0.5t - \frac{0.282\,61}{\alpha}t - 0.15\right)\cdot && (10)\\
&\quad \cdot\left(\alpha\left(\frac{0.325}{\alpha}t - \frac{0.575}{\alpha} - 0.575\,t - 0.172\,5\right) - 0.65t + 1.15\right)
\end{aligned}
$$

Outcome $x_u = F, x_d = H$

$$
\begin{aligned}
\pi_d(F,H) &= (0.025\,\alpha + 0.391\,31t + 0.25\alpha t - 0.25)^2 + && (11)\\
&\quad +1.3\,(0.025\,\alpha - 0.108\,70t + 0.25\alpha t - 0.25)^2 \\
\pi_u(F,H) &= \alpha\begin{pmatrix} \alpha\left(\frac{0.250\,00}{\alpha}t - \frac{0.575}{\alpha} - 0.575\,t - 0.057\,5\right) \\ -0.5t + 1.15 \end{pmatrix}\cdot && (12)\\
&\quad \cdot\left(\frac{0.5}{\alpha} - 0.5t - \frac{0.217\,39}{\alpha}t - 0.05\right)
\end{aligned}
$$

Outcome $x_u = F, x_d = F$

$$\begin{aligned}\pi_d (F,F) &= 1.3\,(0.025\,\alpha + 0.358\,70t - 0.25)^2 + \qquad (13)\\ &\quad + (0.025\,\alpha - 0.141\,31t - 0.25)^2 \\ \pi_u (F,F) &= \alpha \left(\alpha \left(\frac{0.325}{\alpha} t - \frac{0.575}{\alpha} - 0.057\,5\right) - 0.65t + 1.15\right) \cdot \qquad (14)\\ &\quad \cdot \left(\frac{0.5}{\alpha} - \frac{0.282\,61}{\alpha} t - 0.05\right)\end{aligned}$$

In solving the game, the space of parameters is restricted to those values of α and t that are low enough, so that the downstream firm sells positive amounts of the consumer good in each country. In the Appendix, it is shown that the bounds imposed on the parameters are given by:

$$0 < \alpha < \frac{10}{3} \wedge \begin{cases} 0 < t < \frac{2500(10-\alpha)}{39130+25000\alpha} & if \quad \alpha < 0.33229 \\ 0 < t < \frac{(10-3\alpha)2500}{25000\alpha+35869} & if \quad \alpha > 0.33229 \end{cases} \qquad (15)$$

It is easy to check that, for all feasible values of α and t (as defined in 15), the following inequalities hold:

$$\pi_d (H,H) > \pi_d (H,F) \text{ from 7 and 9} \qquad (16)$$

$$\pi_u (F,F) > \pi_u (H,F) \text{ from 14 and 10} \qquad (17)$$

Assuming that the upstream firm locates in Home and that the downstream firm locates in Foreign, the first inequality means that, if firm D deviates to country H, it eliminates the transport cost of the intermediate good and becomes closer to the majority of the consumers, while production costs are kept constant. Also, the second inequality means that, if firm U deviates to country F, it eliminates the transport cost of the intermediate good and minimizes labor costs, while keeping final demand constant.

Together, inequalities 16 and 17 have two different consequences. The first one is that (H, F) is never a Nash equilibrium of locations. The second one is that the best reply correspondence of firm U is completely determined by $\operatorname{sign}[\pi_u(H,H) - \pi_u(F,H)]$, while the best reply correspondence of firm D is also completely determined by $\operatorname{sign}[\pi_d(F,H) - \pi_d(F,F)]$. The equation system

$$\begin{aligned} \pi_u(H,H) - \pi_u(F,H) &= 0 \\ \pi_d(F,H) - \pi_d(F,F) &= 0 \end{aligned} \tag{18}$$

has a unique feasible solution (in the sense of 15), namely

$$\alpha = 0.13043 \tag{19}$$

$$t = 0.2 \tag{20}$$

It is simple to check that

$$\begin{aligned} \pi_d(F,F) &\gtreqless \pi_d(F,H) \text{ iff } \alpha \gtreqless 0.13043 \\ \pi_u(H,H) &\gtreqless \pi_u(F,H) \text{ iff } t \gtreqless 0.2 \end{aligned}$$

Hence, the regions where a different type of locational Nash equilibrium holds are bounded by 15, 19 and 20. These are plotted in Figure 1.

In order to interpret Figure 1, it should be borne in mind that, while t is the transport cost of the consumer good, the product αt is the transport cost of the intermediate good that is required to produce one unit of the consumer good. If

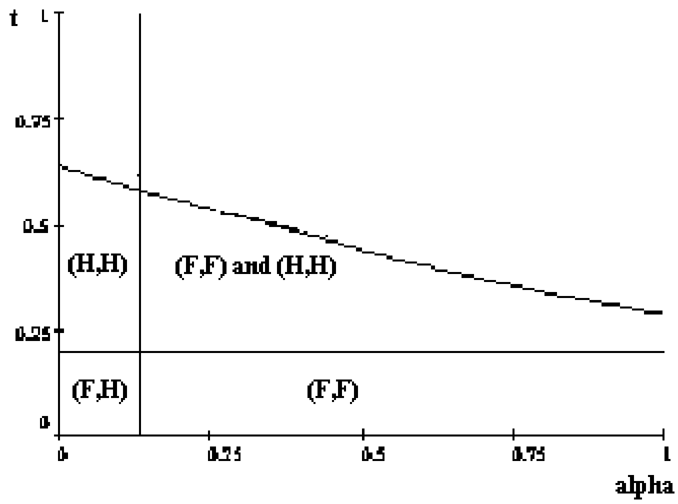

Figure 1: Location equilibria in (α, t) space.

α and t are both low, the transport cost of the intermediate good is very low. Hence, the upstream and the downstream firms choose separate locations (F, H), the former seeking low production costs and the latter seeking high demand. The opposite case is the one where both α and t are high, so that the transport cost of the intermediate good is high. In this case, the firms agglomerate and there are multiple equilibria (H, H) and (F, F), as was stressed by FUJITA (1981): locations do not matter as long as the firms cluster and thus avoid the transport cost of the heavy intermediate good.

On the other hand, if α is low and t is high, the transport cost of the consumer good is high in relation to the transport cost of the input. Hence, locations are driven by demand, so that both firms locate in the country that contains the majority of the consumers. Finally, if α is high and t is low, the transport cost of the intermediate good is high in relation to the transport cost of the consumer good. Hence, locations are driven by the minimization of production costs, so that both firms locate in the low labor cost country F.

3. Concluding remarks

The model presented in the previous section enabled us to explain the trends of the location of vertically-linked firms, whenever transport costs are reduced by an improvement in the transportation system. When transport costs are high, firms cluster in the country with the larger market (although agglomeration can also occur in a peripheral country provided that vertical linkages are high enough). Then, the decrease of transport costs shifts the location of the firms towards the low-cost country. This process has two different possible forms. If the intensity of vertical linkages is low and the industry is labor-intensive (as in the textile industry), the fall of transport costs leads to fragmentation: production is located in the low-cost country, while the distribution and design are located in the larger market. By contrast, sectors where the intensity of input-output relations is high, such as the engineering sectors (as cars, aerospace, pharmaceuticals, electronics), remain agglomerated, but the location of the cluster shifts towards the low-cost country.

As a first step, this paper is based on a numerical example. Its generalization is left for further research.

Appendix: Solution of the price subgames and feasibility condition.

(i) In the case (H, H), the profit functions 4 and 5 become

$$\pi_d(H,H) = 1.3(1-p_h)(p_h-\alpha k)+(1-p_f)(p_f-\alpha k-t) \tag{21}$$

$$\pi_u(H,H) = \alpha[1.3(1-p_h)+(1-p_f)](k-0.3) \tag{22}$$

Maximizing 21 in relation to p_h,p_f, we obtain the prices of the consumer good:

$$p_f = 0.5t+0.5\alpha k+0.5 \tag{23}$$

$$p_h = 0.5\alpha k+0.5 \tag{24}$$

Plugging 23 and 24 into 22 and maximizing the profit function of the upstream firm in relation to the price of the intermediate good, we obtain

$$k=\frac{0.5}{\alpha}-\frac{0.217\,39}{\alpha}t+0.15 \tag{25}$$

Substituting 23, 24 and 25 in the profit functions 22 and 21, we obtain the profit functions in terms of α and t, as given by 8 and 7. The condition of positivity of the outputs sold in the two markets is such that

$$p_f<1\Leftrightarrow t<\frac{2500(10-3\alpha)}{39130} \tag{26}$$

A necessary condition to ensure that this inequality is met is

$$\alpha<\frac{10}{3} \tag{27}$$

(ii). In the case (H,F), the profit functions 4 and 5 become

$$\begin{aligned} \pi_d(H,F) &= 1.3(1-p_h)(p_h-\alpha k-t)+(1-p_f)(p_f-\alpha k) && (28) \\ \pi_u(H,F) &= \alpha[1.3(1-p_h)+(1-p_f)](k-t-0.3) && (29) \end{aligned}$$

Maximizing 28, the delivered prices of the consumer good are obtained

$$\begin{aligned} p_f &= 0.5\alpha k+0.5 && (30) \\ p_h &= 0.5t+0.5\alpha k+0.5 && (31) \end{aligned}$$

Plugging 30 and 31 into the profit function 28 and maximizing this profit function with relation to k, we obtain the price of the intermediate good

$$k = 0.5t+\frac{0.5}{\alpha}-\frac{0.28261}{\alpha}t+0.15 \qquad (32)$$

Substituting 32, 30 and 31 into 29 and 28, we obtain the profit functions in terms of α and t given by 9 and 10.

A sufficient condition so that the downstream firm sells a positive amount of consumer good in each market in the case (H,F) is that the output sold in market H (the distant market) is positive. Given 31 and 32, this condition means that

$$p_h<1 \Leftrightarrow t<\frac{(10-3\alpha)\,2500}{25000\alpha+35869} \qquad (33)$$

A necessary condition so that 33 is fulfilled is

$$\alpha<\frac{10}{3} \qquad (34)$$

(iii) In the case (F, H), the profit functions 4 and 5 become

$$\pi_d(F,H) = 1.3(1-p_h)(p_h-\alpha k)+(1-p_f)(p_f-\alpha k-t) \tag{35}$$

$$\pi_u(F,H) = \alpha[1.3(1-p_h)+(1-p_f)](k-t-0.1) \tag{36}$$

Maximizing 35 we obtain the prices of the consumer good in each country

$$p_f = 0.5t+0.5\alpha k+0.5 \tag{37}$$

$$p_h = 0.5\alpha k+0.5 \tag{38}$$

Plugging 37 and 38 into 36 and maximizing the upstream profit function, the price of the intermediate good is obtained

$$k = 0.5t+\frac{0.5}{\alpha}-\frac{0.217\,39}{\alpha}t+0.05 \tag{39}$$

Substituting 37, 38 and 39 in the profit functions 35 and 36, we obtain the profit functions in terms of α and t, as given by 11 and 12.

A sufficient condition so that the output sold in each market is positive is

$$p_f<1 \Leftrightarrow t<\frac{2500(10-\alpha)}{39130+25000\alpha} \tag{40}$$

A necessary condition so that this inequality is met is

$$\alpha<10 \tag{41}$$

(iv) In the case (F, F), the profit functions 4 and 5 become

$$\pi_d(F,F) = 1.3(1-p_h)(p_h-\alpha k-t)+(1-p_f)(p_f-\alpha k) \quad (42)$$

$$\pi_u(F,F) = \alpha[1.3(1-p_h)+(1-p_f)](k-0.1) \quad (43)$$

Maximizing 42, we find the prices of the consumer good

$$p_f = 0.5\alpha k + 0.5 \quad (44)$$

$$p_h = 0.5t + 0.5\alpha k + 0.5 \quad (45)$$

Plugging these prices into the profit function 43 and maximizing it in relation to k, we obtain the price of the intermediate good

$$k = \frac{0.5}{\alpha} - \frac{0.282\,61}{\alpha}t + 0.05 \quad (46)$$

If we substitute 44, 45 and 46 in the profit functions 42 and 43, we obtain these profit functions in terms of α and t, as given in 13 and 14.

A sufficient condition so that the downstream firm sells a positive amount in each country is that

$$p_h < 1 \Leftrightarrow t < \frac{2500(10-\alpha)}{35870} \quad (47)$$

A necessary condition so that 47 is fulfilled is

$$\alpha < 10 \quad (48)$$

(v) Comparing the bounding conditions 26, 33, 40 and 47 (together with 27, 34, 41 and 48), it is easy to conclude that a sufficient condition so that the downstream firm sells positive outputs in each country is given by

$$0<\alpha<\frac{10}{3}\wedge\begin{cases}0<t<\frac{2500(10-\alpha)}{39130+25000\alpha} & if \quad \alpha<0.33229\\ 0<t<\frac{(10-3\alpha)2500}{25000\alpha+35869} & if \quad \alpha>0.33229\end{cases}$$

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